

## Loschmidt echo and the local density of states

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(Received 7 August 2009; published 26 October 2009)

Loschmidt echo (LE) is a measure of reversibility and sensitivity to perturbations of quantum evolutions. For weak perturbations its decay rate is given by the width of the local density of states (LDOS). When the perturbation is strong enough, it has been shown in chaotic systems that its decay is dictated by the classical Lyapunov exponent. However, several recent studies have shown an unexpected nonuniform decay rate as a function of the perturbation strength instead of that Lyapunov decay. Here we study the systematic behavior of this regime in perturbed cat maps. We show that some perturbations produce coherent oscillations in the width of LDOS that imprint clear signals of the perturbation in LE decay. We also show that if the perturbation acts in a small region of phase space (local perturbation) the effect is magnified and the decay is given by the width of the LDOS.

DOI: [10.1103/PhysRevE.80.046216](https://doi.org/10.1103/PhysRevE.80.046216)

PACS number(s): 05.45.Mt, 05.45.Ac, 05.45.Pq, 03.65.Yz

### I. INTRODUCTION

Reversibility and sensibility to perturbations of quantum systems are at the heart of fields of research as vast as quantum computation, quantum control, and coherent transport [1–3]. The interest in these subjects has greatly increased due to the development of experimental techniques that enable the manipulation of a great number of quantum systems from photons to mesoscopic devices [4,5].

The suitable magnitude for measuring the stability of the quantum motion, as well as its irreversibility, is the Loschmidt echo (LE) [6,7], defined as,

$$M(t) = |\langle \psi_0 | \exp(iHt) \exp(-iH_0t) | \psi_0 \rangle|^2, \quad (1)$$

where  $\psi_0$  is the initial state,  $H_0$  is the nonperturbed Hamiltonian, and  $H$  is the slightly perturbed one. For convenience, we set  $\hbar=1$  throughout the paper. The initial state  $\psi_0$  is usually a Gaussian wave packet and it frequently refers to an average taken over an ensemble of initial states randomly localized in phase space. The LE has been extensively studied in recent years and different time and perturbation strength regimes were shown to exist [8–11]. As a function of time, this magnitude has three well-known regimes. For very short times, it is parabolic or Gaussian, as the perturbation theory is valid to first order [12]. This transient regime is followed by a decay that is typically exponential in chaotic systems [8]. Finally, the LE finds a long-time saturation at values inversely proportional to the effective size of the Hilbert space [10,13].

As a function of the strength of the perturbation, the decay of the LE has mainly three different behaviors [7,8]. When the perturbation is very small, in which a typical matrix element  $W$  of the perturbation is smaller than the mean level spacing  $\Delta$ , the decay is Gaussian until  $M$  reaches its asymptotic values. If  $W > \Delta$ , this regime has an exponential decay, with decay rate given by the width  $\sigma$  of the local density of states (LDOS). This is usually called Fermi golden rule regime (FGR). Finally, when  $\sigma > \lambda$ , with  $\lambda$  the mean

Lyapunov exponent of the classical system, the regime becomes independent of the perturbation and the decay rate is given by  $\lambda$ . Although this Lyapunov regime seems to be universal as the intensive numerical studies have shown in the literature [9,10], recent works have found a nonuniform behavior of the decay rate as a function of the perturbation strength. This was observed in an echo spectroscopy experiment on ultracold atoms in optical billiards and in theoretical studies of the kicked rotator, the sawtooth map, and Josephson flux qubits [14–18]. Similar qualitative behavior was shown in Ref. [19] for local perturbations. In this case the authors found an oscillating regime of the decay of the LE around the value of the classical escape rate.

Our main aim in this paper is to study the nonuniform decay rate of the LE in the strong perturbation regime mentioned before. We show that the origin of such behavior is related to the interplay between the width LDOS and the Lyapunov exponent. The LDOS is a measure of the action of a perturbation over a system and it gives the spread of the states of the unperturbed Hamiltonian in the bases of the perturbed one. In general, the width of the LDOS has a quadratic growth when the strength of the perturbation is small and, for increasing strength, the LDOS has a constant plateau  $\bar{\sigma}$  or an oscillating behavior around  $\bar{\sigma}$ . This regime, which does not present a steady growth, is related to the finite number of states of the Hilbert space linked by the perturbation [21]. We show that if  $\bar{\sigma} \gg \lambda$  we obtain the expected result that the decay rate of the LE is given by  $\lambda$  when the perturbation is strong enough. However, if  $\bar{\sigma} \leq \lambda$ , the nonuniform behavior of the width of the LDOS around  $\bar{\sigma}$  seems to be imprinted in the decay rate of the LE, that is, the decay of the echo shows fluctuations that can mask the Lyapunov decay. This unexpected result can be understood using the semiclassical dephasing representation of the LE [22]. We also consider local perturbations, that is, the case where the perturbation is applied to a window of the positions. These perturbations allow changing the width of the LDOS while the Lyapunov exponent remains fixed as the region of the applied perturbation is varied. Moreover, the use of local perturbations is motivated by recent semiclassical work of Ref. [19] and by the fact that local perturbations are more amenable for labo-

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ratory experiments [20]. In this case we show that the smaller the slice of the perturbed region is, the more similar the decay rate of the LE and the width of the LDOS are. Besides, we have explicitly exhibit the predicted oscillations of the decay rate as a function of the strength of a local perturbation obtained in the semiclassical study of Ref. [19].

The model that we use in our study is the cat map, a paradigmatic system in quantum chaos that is attracting increasing interest in the area of quantum computation [23,24]. This system is completely chaotic and is very well suited to our work. It can be perturbed in different ways and the value of  $\lambda$  can be chosen at will. Moreover, the perturbations can be easily applied to all (global) or a section (local) of the phase space.

The paper is organized as follows. Section II describes the main characteristics of the classical and quantum cat maps. We introduce the perturbations used in our work and the LDOS, a measure of the perturbation action over the system. In Sec. III we show the results of the decay of the LE for cat maps with different Lyapunov exponents using different perturbations. In this section we show the relation between the decay of the LE and the width of the LDOS. In Sec. IV we make a similar study but for local perturbations. Finally, Sec. V is devoted to final remarks and conclusions.

## II. PERTURBED CAT MAPS

Torus maps are canonical examples of classical chaotic systems. In this work, we will focus on cat maps, which are linear automorphisms of the torus that exhibit hard chaos. Anosov's theorem establishes that they are structurally stable, that is, that orbits of a slightly perturbed map are conjugated to those of the unperturbed map by a homeomorphism. To make the paper self-contained we present in this section the main aspects of the classical and quantum perturbed cat maps that are important for our study.

### A. Classical system

Cat maps for a two-dimensional torus considered in a unit periodic square are represented by matrices acting on the coordinates

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} \pmod{1}. \quad (2)$$

We take integer entries in the matrix  $G$  to ensure continuity, and also  $\text{Tr}[G] > 2$  and  $\det[G] = 1$  since the map is hyperbolic and conservative.

The maximal logarithm of the eigenvalues of  $G$  defines its Lyapunov exponent, quantity that characterizes the rate of separation of infinitesimally close trajectories. We consider the following cat maps:

$$G_1 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \quad G_2 = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad G_3 = \begin{pmatrix} 4 & 1 \\ 15 & 4 \end{pmatrix}. \quad (3)$$

The corresponding Lyapunov exponents are  $\lambda_1 = \log(\frac{1}{2}(3 + \sqrt{5})) \approx 0.96$ ,  $\lambda_2 = \log(2 + \sqrt{3}) \approx 1.32$ , and  $\lambda_3 = \log(4 + \sqrt{15}) \approx 2.06$ , respectively.

Now we introduce a perturbation of the cat map that is a shear in momentum that depends on coordinates and we take its linear part equal to zero at the origin, according to [26],

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = G \begin{bmatrix} q \\ p + \epsilon(q) \end{bmatrix}, \quad (4)$$

where in particular, we considered,

$$\epsilon(q) = \frac{k}{2\pi} [\cos(2\pi q) - \cos(4\pi q)], \quad (5)$$

with  $k$  being the strength of the perturbation. We note that  $k < 0.11$  for the perturbation strength to satisfy the Anosov theorem [26]. We have also used a more general perturbation that is a shear in momentum and position (see below). We notice that the Lyapunov exponent of all the cat maps that we have considered does not change significantly when the mentioned perturbations are taken into account.

### B. Quantum system

Quantization of perturbed cat maps has been an important step in the quantum chaos studies because these simple systems allow the understanding of many manifestations of chaos in quantum systems [25–27]. The wave function should be periodic in both position and momentum representation due to the periodic nature of the torus. Consequently, the wave functions are periodic combinations of delta functions. If in the coordinate representation the wave function has a period  $\Delta q$  with spacing  $\Delta q/N$ , then in the momentum representation the period is  $\Delta p = 2\pi\hbar N/\Delta q$  with spacing  $2\pi\hbar/\Delta q$ . If  $\Delta p = \Delta q = 1$ , it follows that  $1 = 2\pi\hbar N$ . Then, we have a Hilbert space of  $N$  dimension for a fixed value of  $\hbar$ . As  $N$  takes increasing values, we reach the semiclassical limit.

The propagator is obtained from the quadratic generating function and it can be written as a  $N \times N$  matrix acting over a vector with components being the amplitudes of each delta function. Up to a phase, it results to

$$U^G(q', q) = \sqrt{\frac{N}{ig_{12}}} \exp \left[ \frac{i\pi N}{g_{12}} (g_{11}q^2 - 2q'q + g_{22}q'^2) \right]. \quad (6)$$

The phase factor is equal to the unity if  $g_{12} = 1$ . It can be shown that only cat maps with odd diagonal and even anti-diagonal (or vice versa) can be quantized [25]. In the case of  $G_1$  the number  $N$  of states of the Hilbert space must be even [28], a requirement that we have satisfied.

In order to quantize the perturbed map, we use the fact that it can be written as a composite map. If  $P$  is the simple shear, the quantized map is thus defined by composing the evolution operators for the cat map and the shear in the same order as the classical propagator,

$$U = U^G U^P, \quad (7)$$

with  $U^P = \exp[i2\pi NS_p(q)]$  and  $\frac{-dS_p(q)}{dq} = \epsilon(q)$ , so,

$$S_p(q) = \frac{k}{4\pi^2} \left( \sin(2\pi q) - \frac{1}{2} \sin(4\pi q) \right). \quad (8)$$

Now we define the scaled strength,

$$\chi \equiv \frac{k}{2\pi\hbar} = kN. \quad (9)$$

Finally, we have the propagator for the perturbed cat map,

$$U(q', q) = \sqrt{\frac{N}{ig_{12}}} \exp \left[ \frac{i\pi N}{g_{12}} (g_{11}q^2 - 2q'q + g_{22}q'^2) \right] \times \exp \left[ \frac{i\chi}{2\pi} \left( \sin(2\pi q) - \frac{1}{2} \sin(4\pi q) \right) \right]. \quad (10)$$

To introduce more general perturbations, we consider shears in both momentum and positions [27]. Equation (10) shows that a shear in momentum is diagonal in the coordinate representation. For a shear in position, we can construct the propagator by changing to momentum coordinates and finding a diagonal matrix in this representation. The change of basis matrix is the discrete Fourier transform  $F$ , where

$$F_{lj} = \frac{1}{\sqrt{N}} \exp \left( \frac{-2\pi ilj}{N} \right). \quad (11)$$

In this work, we use the following shears in momentum and positions:

$$U = U^P U^G F^+ U^Q F, \quad (12)$$

where  $U^Q = \exp \left\{ \frac{i\chi}{2\pi} \left[ \cos(6\pi p) - \frac{1}{2} \sin(4\pi p) \right] \right\}$ .

### C. Local density of states

The action of a perturbation on the eigenstate of a quantum system can be described by the LDOS. Let  $\phi_j(k)$  and  $\psi_j(k)$  be the eigenphases and eigenfunctions of a perturbed cat map ( $k$  is the strength of the perturbation and  $j=1, \dots, N$ ). For an eigenstate  $i$  at  $k_0$ , considered as the unperturbed eigenstate, the LDOS is defined as follows:

$$\rho_i(\phi, \Delta k) = \sum_j |\langle \psi_j(k) | \psi_i(k_0) \rangle|^2 \delta(\phi - [\phi_j(k) - \phi_i(k_0)]), \quad (13)$$

where  $\Delta k = k - k_0$ . The initial strength  $k_0 \neq 0$  is needed to remove the peculiar non generic behavior of the unperturbed cat map [25,26]. To avoid a dependence on the particular characteristics of the unperturbed state  $i$  we always make an average over this state.

The width  $\sigma$  of the LDOS gives us an idea of how many states in the base of the unperturbed system are needed to describe a state of the perturbed one. So, it is a good measure of the perturbation action over the system. This quantity can be measured in different ways [29]. In our case, we take the half distance around the mean value that contains the 70% of the probability [29].

We compute the width  $\sigma$  for both perturbations introduced before (see Fig. 1). In the main plot of Fig. 1,  $\sigma$  is shown as a function of  $\chi$  when the cat map  $G_2$  is perturbed with a

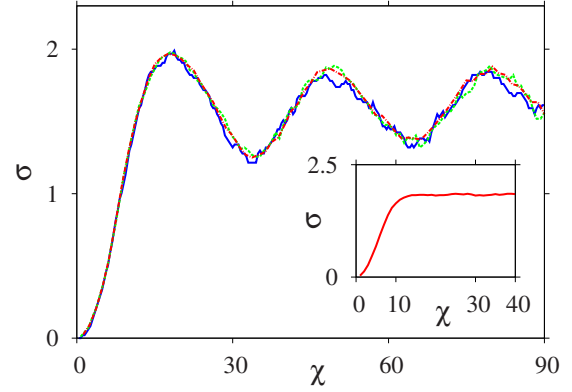


FIG. 1. (Color online) Width  $\sigma$  of LDOS as a function of the scaled perturbation strength  $\chi$  for  $N=300$  (green dashed line),  $N=600$  (blue solid line), and  $N=1200$  (red dotted-dashed line). The perturbation is a shear in positions for the cat map given by  $G_2$ . We have obtained the same results for  $G_1$  and  $G_3$ . It is clear that  $\sigma$  does not depend on the size of the Hilbert space and exhibits an oscillatory behavior for strong perturbations. Inset:  $\sigma$  vs  $\chi$  for the same cat map perturbed with shears in both momentum and positions. It remains steady for strong perturbations.

shear in momentum [Eq. (10)]. The initial value of the strength is  $k_0=0.02$  and the width is computed for  $N=300$ ,  $N=600$ , and  $N=1200$ . We clearly see that the width of the LDOS does not depend on the number of states of the Hilbert space. We also notice that  $\sigma$  has an oscillatory behavior for  $\chi \geq 15$ . In the small perturbation regime ( $\chi \leq 15$ ), we see the quadratic behavior that can be obtained in a perturbative way [30]. For shears in momentum and position [Eq. (12)], we also compute  $\sigma$  as a function of  $\chi$  for  $N=1200$  [see the inset of Fig. 1]. In contrast to the previous results, the width  $\sigma$  is constant for  $\chi \geq 15$ . The quadratic perturbative regime is present again for small perturbations.

From the results of Fig. 1, it is clear that the oscillatory behavior is related to the type of perturbation considered. In order to illustrate this behavior in more detail, we show the LDOS [Eq. (13)] as a function of the scaled strength  $\chi$  for the considered perturbations. We have made an average over 50 unperturbed states. Figure 2(a) is a density plot of  $\rho(\Delta\phi, \chi)$  when the cat map  $G_2$  is perturbed with a simple shear in momentum [Eq. (10)] and, in Fig. 2(b), with both shears in momentum and positions [Eq. (12)]. In the first case [see Fig. 2(a)] and for  $\chi < 10$ , the distribution has a big peak with Lorentzian shape whose width becomes wider until its tails reach the limits of the Hilbert space. Then two small peaks appear at  $\chi \approx 10$ . These peaks move to the border and then come back to the center from both sides. That denotes the coherence given by this kind of perturbation. In contrast, when the shear is more general, the distribution is almost uniform for  $\chi > 10$  and it lacks any coherence. We remark that the results that are shown in Figs. 1 and 2 for  $G_2$  are almost the same for the maps  $G_1$  and  $G_3$ .

### III. LOSCHMIDT ECHO

This section is devoted to the study of the decay rate  $\Gamma$  of the exponential decay of the LE. We compute  $\Gamma$  as a function



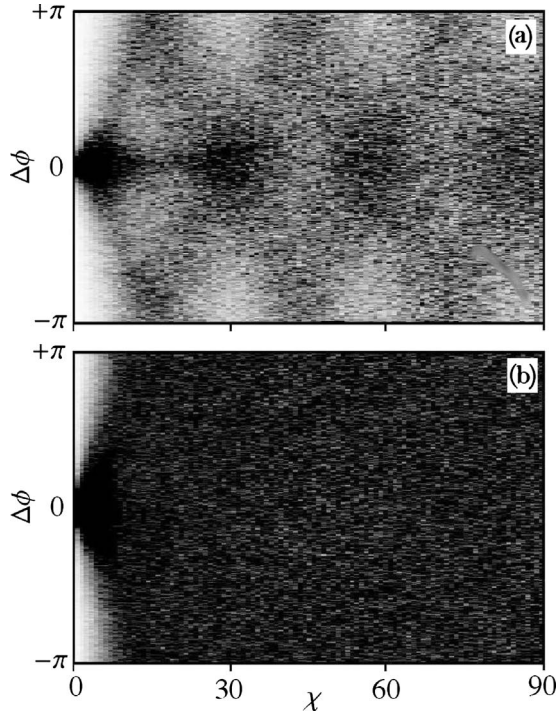


FIG. 2. Density plots of the LDOS  $\rho(\Delta\phi, \chi)$  as a function of  $\chi$ . An average over 50 unperturbed states was done. (a) The perturbation is a simple shear in momentum. We can notice the coherence that is responsible for the oscillations found in the width of the LDOS. (b) The shear is in momentum and coordinates. In this case, no coherence is observed.

of  $\chi$  for the cat maps  $G_1$ ,  $G_2$ , and  $G_3$  perturbed by a shear in momentum [Eq. (10)] and by shears in momentum and position [Eq. (12)]. We have used  $N=2000$  in all the computations of this section. This value of the number of states of the Hilbert space allows for a good fit of the decay of the LE due to the fact that the long-time saturation occurs at a value inversely proportional to the effective size of the Hilbert space of the system. The initial states are coherent states randomly localized in phase space. An average over 200 initial states was made.

Figure 3 shows  $\Gamma$  as a function of  $\chi$  for  $G_1$  [panel (a)],  $G_2$  [panel (b)], and  $G_3$  [panel (c)] perturbed by a shear in momentum. The width  $\sigma$  of the LDOS (solid lines) and the value of the Lyapunov exponents (dotted lines) are also plotted. In the insets we show the behavior of  $\Gamma$  in the weak perturbation regime. It is clear that for weak perturbations the decay rate of the LE and the width of the local density of states are equal: both of them follow the quadratic behavior given by the Fermi Golden Rule. For strong perturbations,  $\Gamma$  reaches the value of the Lyapunov exponent in the case of  $G_1$ , when the oscillations of the width of LDOS occur at greater value than  $\lambda_1$  [Fig. 3(a)]. However, in the last two cases, it exhibits an oscillatory behavior that is evidently related to  $\sigma$  [Figs. 3(b) and 3(c)]. In the case of the map given by  $G_2$ ,  $\lambda_2$  is less than the limit reached by  $\sigma$  and we have just mentioned that we can observe the influence of the width of the local density of states in  $\Gamma$ . For  $G_3$ ,  $\lambda_3$  is greater than that limit, and the influence seems to be stronger. In this case, we see in Fig. 3(c) that the decay  $\Gamma$  near the first peak

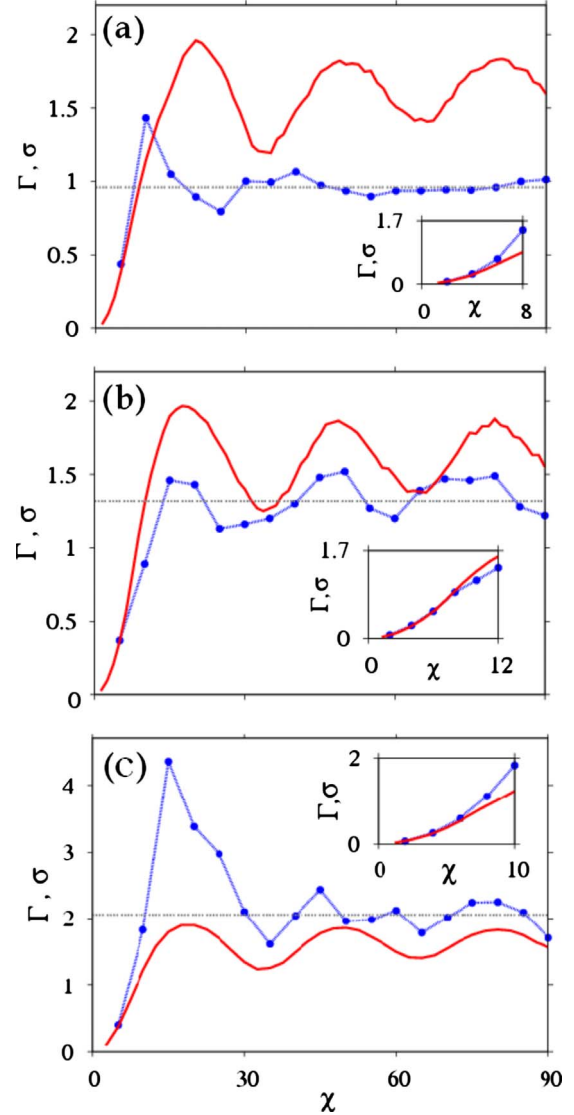


FIG. 3. (Color online) Decay rate  $\Gamma$  of the LE (full circles) of a cat map perturbed with a shear in momentum as a function of the scaled perturbation strength  $\chi$ . The width  $\sigma$  of the LDOS (solid lines) and the Lyapunov exponent (dotted lines) are also plotted. (a) The cat map is given by  $G_1$ . In this case  $\lambda_1 < \sigma$  for  $\chi > 15$ . Here the LDOS do not influence the LE decay in the strong perturbation regime. (b) The cat map is given by  $G_2$ . In this case  $\lambda_2$  is similar to the strong perturbation limit of  $\sigma$ . Now we clearly see that the oscillations of the LE decay rate are strongly related to the ones of  $\sigma$ . (c) The cat map is given by  $G_3$ . In this case  $\lambda_3 > \sigma$  for strong perturbations. The oscillations exhibited by the LE decay rate follow the  $\sigma$  ones with a larger amplitude. In the insets the weak perturbation regime can be clearly seen (FGR regime).

of  $\sigma$  at  $\chi \approx 15$  is strongly enhanced and the decay rate reaches to  $\Gamma \approx 2\lambda$ . The  $2\lambda$  decay was studied using the uniform semiclassical approach in Ref. [16]. This behavior deserves further investigation [31].

However, for the cat map given by  $G_3$ , there are some points for  $45 < \chi < 60$  that seem to be misplaced. The reason for this disagreement is that the fast decrease in the echo caused by the large Lyapunov exponent makes  $\Gamma$  very difficult to determine. As we want to verify the behavior found

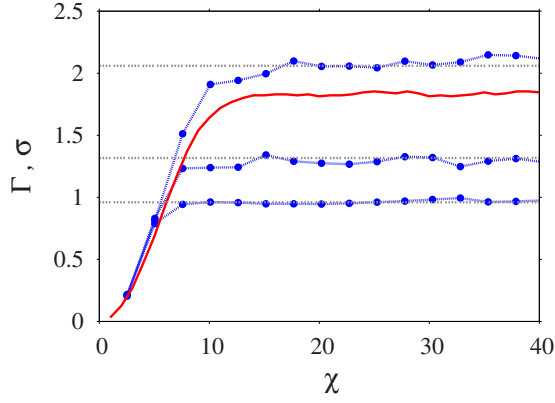


FIG. 4. (Color online) Decay rate  $\Gamma$  of the LE (full circles) of a cat map perturbed by a shear in momentum and position as a function of the scaled perturbation strength  $\chi$ . The width  $\sigma$  of the LDOS (solid lines) and the Lyapunov exponent (dotted lines) are also plotted. The cat map is given by  $G_1$  (bottom),  $G_2$  (middle) and  $G_3$  (top). See text for details.

for  $\Gamma$  when  $\lambda$  is greater than the limit reached by  $\sigma$ , we repeated the calculations for the cat map given by

$$G_4 = \begin{pmatrix} 8 & 1 \\ 63 & 8 \end{pmatrix}, \quad (14)$$

whose Lyapunov exponent is  $\lambda_4 = \log(8 + 3\sqrt{7}) \approx 2.77$ . The results were completely analogous. To improve the fitting it would be necessary to run these calculations for greater values of  $N$ , so the decay rate could be better distinguished but it would demand a really long computational time because the computational time goes as  $N^2$ . In the next section we will show that local perturbations allow to study the case in which the Lyapunov exponent is much smaller than the width of the LDOS.

In order to confirm that the oscillatory behavior observed in the decay rate of the LE is due to the influence of the oscillations of the width of LDOS, we also make the same study as before for the perturbation [Eq. (12)] that does not have any oscillations [see the inset of Figs. 1 and 2(b)]. In Fig. 4 we show  $\Gamma$  for  $G_1$ ,  $G_2$ , and  $G_3$ , together with  $\sigma$ , for this general perturbation. It is clear that as  $\sigma$  has no oscillations, neither does  $\Gamma$ , and this behavior is the same for the different cat maps considered. In all cases, they reach the Lyapunov value for strong perturbations. The last observation supports our conclusion about the influence of the oscillations of the width of LDOS over the decay of the LE in the Lyapunov regime.

Now, we can go one step further by giving a semiclassical interpretation of our results using the dephasing representation of the LE by Vanicek [22]. The LE is found to be

$$M(t) = |O(t)|^2 = \left| \left( \frac{A^2}{\pi \hbar^2} \right)^{d/2} \int d^d p' \times \exp \left[ \frac{i}{\hbar} \Delta S(\mathbf{r}(\mathbf{r}_0, \mathbf{p}', t), \mathbf{r}_0, t) - (\mathbf{p}' - \mathbf{p}_0)^2 \frac{A^2}{\hbar^2} \right] \right|^2, \quad (15)$$

where  $d$  the number of degrees of freedom and  $\mathbf{r}_0$  is the

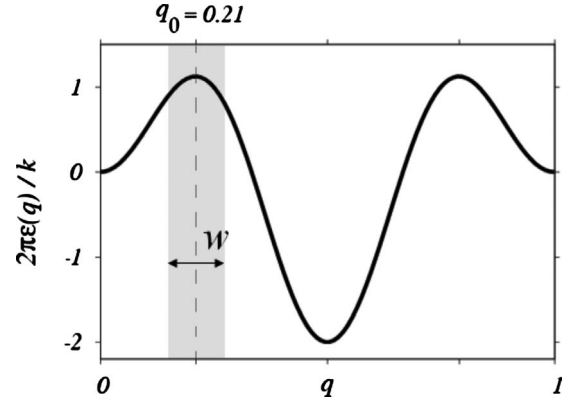


FIG. 5. Schematic figure showing the local perturbation that was used. The scaled shear  $[2\pi\epsilon(q)/k]$  ( $k$  is the strength of the perturbation) is plotted as a function of the  $q$  coordinate.  $q_0$  indicates the center of the interval that was perturbed and  $w$  its width.

center of the initial Gaussian wave packet, with dispersion  $A$  and an average momentum  $\mathbf{p}_0$ .  $\Delta S(\mathbf{r}, \mathbf{r}_0, t)$  stands for the difference of action of the perturbed and unperturbed orbits. Now, when the shear caused by the perturbation is an integer number of  $1/N$  (the grid of the quantum phase space), the interference between the perturbed and the unperturbed orbits, given by the difference of action, is constructive. Otherwise, it is destructive. So, when the shear is in momentum, that condition is satisfied for certain values of  $k$ , and consequently we observed oscillations with a wavelength proportional to  $1/N$ . The wavelength of the oscillations of  $\sigma$  would be, therefore,  $4\pi^2/N$  in units of  $k$  [Eq. (8)], as we can verify in Fig. 1, where it is approximately  $4\pi^2$  in units of  $\chi$ .

On the contrary, when the shear is in momentum and positions, the interference condition cannot be satisfied simultaneously and the oscillations disappear.

#### IV. LOSCHMIDT ECHO FOR LOCAL PERTURBATIONS

Perturbations to quantum systems are generally local when they come from experiments [20]. This means that understanding of the general behavior of LE for local perturbation is an important issue. Recently, Goussev and co-workers have presented a semiclassical theory for the decay rate of the LE for local perturbations in billiards systems [19]. They show that this kind of perturbations introduces an oscillatory behavior of the decay rate of the LE around the classical escape rate. This quantity corresponds to the probability to escape from the billiard when the classical particle hits the region of the boundary where the perturbation is applied.

So far, we have shown in previous sections that the characteristics of the perturbation modify the behavior of the width of the LDOS and that they shape the decay rate of the LE. With this in mind we have looked for that dependence in the case of local perturbations. Furthermore, we are interested in the behavior of the decay rate of the LE as a function of the width of the perturbed region. With this aim, we have applied the already introduced shear in momentum to a window of coordinates of the phase space. We took the coordi-

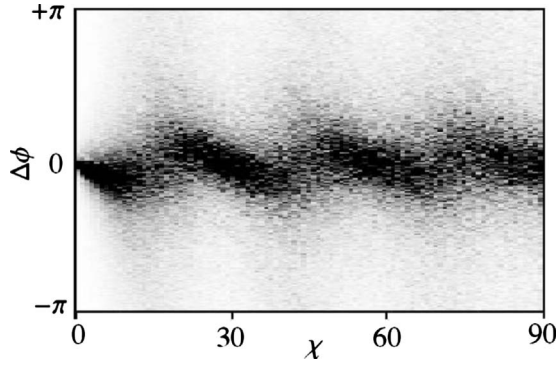


FIG. 6. Density plot of the LDOS  $\rho(\Delta\phi, \chi)$  as a function of the strength  $\chi$  when the perturbation is a simple shear in momentum applied in a region of width  $w=0.4$  (see Fig. 5). It can be clearly seen that it is restricted to the perturbed phases and that it has a coherent behavior.

nate in which the function that describes the shear  $\epsilon(q)$  [Eq. (5)] has one of its maxima as the center of the interval to be perturbed. This is shown in Fig. 5, in which we plot the scaled  $\epsilon(q)$  and the corresponding interval that is perturbed.

Let us first see the influence of a local perturbation in the behavior of LDOS. In Fig. 6 we exhibit the distribution  $\rho(\Delta\phi, \chi)$  when the perturbation is applied in a region of  $w=0.4$ . It is clear that the distribution is bounded and we can see a coherent behavior similar to the one observed in Fig. 2(a). We notice that  $\rho(\Delta\phi, \chi)$  does not present the same symmetry as around  $\Delta\phi=0$  as is observed in Fig. 2(a) when the perturbation is global. This is related with the position  $q_0$  as center of the perturbed region [31].

In Figs. 7 and 8 we show the behavior of the decay rate of the LE as a function of the strength  $\chi$  and the width  $w$  of the local perturbation. All the computations in this section were done with  $N=800$  (the total size of the Hilbert space). We have considered several widths where the

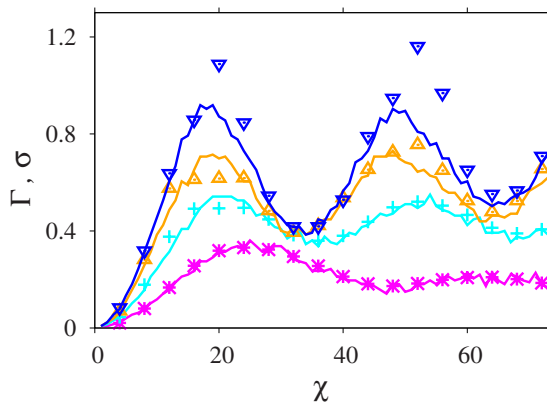


FIG. 7. (Color online) Decay rate  $\Gamma$  of the LE of a cat map perturbed locally with a shear in momentum as a function of the scaled perturbation strength  $\chi$ . The symbols correspond to the following width of the perturbed region:  $\nabla$  ( $w=0.4$ ),  $\Delta$  ( $w=0.3$ ),  $+$  ( $w=0.2$ ), and  $*$  ( $w=0.1$ ). The width  $\sigma$  of the LDOS is also plotted with solid lines. It is noticeable that the smaller  $w$  is, the more similar to  $\sigma$   $\Gamma$  is. In all the cases considered, the oscillations are equal for both magnitudes.

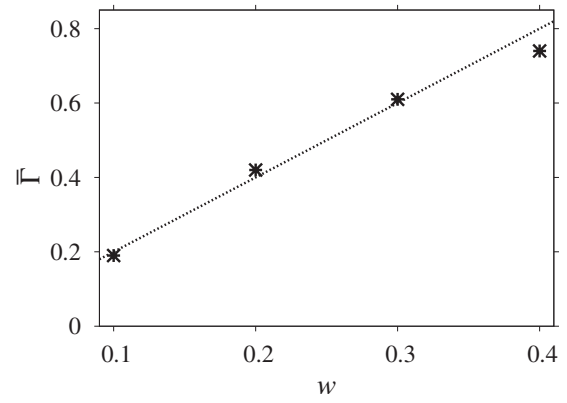


FIG. 8. Mean value  $\bar{\Gamma}$  of decay rate of the LE as a function of the width  $w$  of the applied local perturbation. The function  $2w$  is also plotted with dotted line. See text for details.

perturbation was applied. As an example, in Fig. 7 we show the decay rate  $\Gamma$  of the LE with symbols for:  $\nabla$  ( $w=0.4$ ),  $\Delta$  ( $w=0.3$ ),  $+$  ( $w=0.2$ ), and  $*$  ( $w=0.1$ ). We notice that, although the oscillatory behavior is present for all the values of  $w$  considered, its amplitude is smaller when  $w$  decreases. Moreover, the value around the oscillation takes place also becomes smaller as  $w$  becomes thinner. To conclude the analysis, we also compute the width  $\sigma$  of the LDOS for all the different apertures considered. The width  $\sigma$  is shown in Fig. 7 with solid lines. It is clear that for  $w=0.1$  and  $w=0.2$ , the behaviors of  $\Gamma$  and  $\sigma$  are identical. That is, the decay rate of the LE is given by the width of the LDOS. This result is in agreement with the semiclassical study for billiard systems of Ref. [19]. For bigger  $w$  ( $w=0.3$  and  $w=0.4$  in our case), the oscillations are alike, but its amplitude is not so similar. So, in the case of a local perturbation, the influence of the LDOS width over  $\Gamma$  is enhanced.

In Fig. 8 we show the mean value of the decay rate of the LE  $\bar{\Gamma}$  after the quadratic growth [ $\chi > 20$ , see Fig. 7] as a function of the width  $w$  of the perturbation. We can clearly see that  $\bar{\Gamma} \approx 2w$  for  $w < 0.4$ . We remark that our result agrees with the one obtained for billiard systems in Ref. [19] because the width  $w$  corresponds to the classical escape rate in our system.

### V. FINAL REMARKS

In this work we have studied the decay of the LE for strong perturbations, that is, the regime after the quadratic growth of the decay (FGR regime). We have shown that the expected monotonic regime in which the decay is given by the Lyapunov exponent of the classical system is not universal. In fact, the behavior of the LE in this regime depends on the ratio between the mean value of the width of LDOS  $\bar{\sigma}$  and the value of the Lyapunov exponent.

The general idea is the following. When a system is perturbed and the perturbation affects a finite number of states of the Hilbert space, instead of growing monotonically, the width of the LDOS reaches a plateau  $\bar{\sigma}$  or a nonuniform regime around  $\bar{\sigma}$ . If  $\bar{\sigma}/\lambda \gg 1$ , the decay rate of the LE is



given by  $\lambda$  and does not depend on the perturbation. But, if  $\bar{\sigma}/\lambda \leq 1$  and the width  $\sigma$  of the LDOS has a nonuniform behavior, a nonmonotonicity is imprinted in the decay of the LE. This regime was recently reported in the literature [14–18] but not the conditions for it to happen.

We have illustrated the previous ideas using cat maps with different Lyapunov exponents and perturbations. When the cat map is perturbed with a simple shear in momentum, we have found pronounced oscillations of the width of the LDOS, which is a consequence of the coherence of perturbed and unperturbed orbits that this kind of perturbation introduces. Significantly, we have shown that this oscillating regime was also present in the decay rate of the LE when the value around the LDOS width oscillates is close enough to the Lyapunov exponent of the classical system. Moreover, when the perturbation is more general, the coherence between orbits is destroyed, so the width of the LDOS reaches a plateau and so does the LE decay.

We have also studied the decay of the LE in cat maps when the perturbation is local [19]. The important point in

this case is that varying the width of the region in which the perturbation is applied, we can change the width of the LDOS while the Lyapunov exponent remains fixed. With this in mind, we have perturbed a range of coordinates with a simple shear in momentum. We have found that the decay rate is almost identical to the width of the LDOS when the range of the perturbed coordinates is very small. Moreover, we have shown that the amplitude of the oscillations of the LDOS decreases with the width of the applied perturbation and the mean value is given by the classical decay rate.

## ACKNOWLEDGMENTS

The authors acknowledge the support from CONICET (Grant No. PIP-6137), UBACyT (Grant No. X237), and ANPCyT. We would like to thank Ignacio García Mata, Pablo Tamborenea and Eduardo Vergini for useful discussions.

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